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Tight Bounds on the Ergodic Capacity of Cooperative Analog Relaying with Adaptive Source Transmission Techniques

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ABSTRACT *— Tight bounds for the Shannon capacity of amplify-and-forward cooperative diversity networks are derived for three different adaptive source transmission policies in a myriad of fading environments: (i) constant power with optimal rate adaptation (ORA); (ii) optimal joint power and rate adaptation (OPRA); and (iii) fixed rate with truncated channel inversion (TCIFR). Our unified framework based on the moment-generating function (MGF) approach allows us to gain insights as to how fade distributions and dissimilar fading statistics across the distinct communication links will affect the Shannon capacity, without imposing any restrictions on the fading parameters.*

Index Terms — wireless link adaptation, cooperative diversity, channel capacity

I. INTRODUCTION

While multiple-input-multiple output (MIMO) architectures could drastically improve the range and reliability of beyond line-of-sight and/or over-the-horizon communication links without increasing transmit-power – the cost, weight, and poor aero-dynamics of antenna arrays may prohibit their use on both unmanned vehicles and dismounted tactical war-fighters. In multi-vehicle cooperative operations, networked nodes in a tight cluster may coordinate both their transmissions and/or receptions to mimic a space-time processing system as if they were part of a single antenna array platform (e.g., [1]-[2]). Thus cooperative airborne networking could significantly increase the range and reliability of a long-haul inter-cluster communication, thereby improving platform endurance with enhanced LPI/LPD capability, without using an antenna array. Several standardization groups such as IEEE 802.16 and IEEE 802.11 have also incorporated cooperative relaying into their emerging wireless standards (e.g., Mobile Multihop Relaying Group has defined a multihop relay architecture in the baseline IEEE 802.16j standard).

Although an intermediate (relay) node in a cooperative wireless network may either amplify what it receives (in case of amplify-and-forward relaying) or digitally decodes, and re-encodes the source message (in case of decode-and-forward relaying) before re-transmitting it to the destination node, we shall focus on the amplify-and-forward (analog) relaying scheme in this article because it does not require "sophisticated" transceivers at the relays. Nevertheless, our framework is also applicable for digital relaying once the MGF of signal-to-noise ratio (SNR) is found or available. While this protocol can achieve full diversity using a virtual antenna array, there is a loss of spectral efficiency due to its inherent half-duplex operation. This penalty could be "recovered" to some extent by combining the cooperative diversity with a link adaptation mechanism wherein the power level, signal constellation size, coding rate or other transmission parameters are adapted autonomously in response to fluctuations in the channel conditions. Besides, fixed transmission methods that are designed to provide the required quality of service in the "worst-case" scenario are very inefficient when better channel conditions prevail.

But the art of adaptive link layer in a cooperative wireless network is still in its infancy especially when optimized in a cross-layer design paradigm. Majority of the literature on cooperative diversity are limited to both fixed signaling rate and/or constant transmit power for all communication nodes. For instance, while [3]-[5] have studied the problem of optimal power allocation in a cooperative wireless network, source rate-adaptation [6] was not considered, and more critically its solution requires the knowledge of channel side information (CSI) of all links. Motivated by these observations, [7] derived bounds for the Shannon capacity of a non-regenerative link-adaptive cooperative diversity system with limited CSI, in which the rate and/or power level at the source node is adapted according to the channel condition (i.e., only feedback of the effective SNR at the destination node is required to be available at the source node) while the relays simply amplify and forward the signals. However, their analysis is limited to Rayleigh fading. But in an airborne platform, it is much more reasonable to model the channel gain of each link as a Nakagami-m or a Rice random variable (due to the increased likelihood of the presence of a strong specular component).

The main objective of this work to develop a unified framework based of the MGF method for evaluating the ergodic capacity of cooperative relaying systems with adaptive source transmission policies (since the MGF of the end-to-end SNR is much easier to compute or may be readily available). This is of significant interest because several authors [8]-[10] have recently argued that although the MGF approach has been successfully and extensively applied for evaluating the performance of wireless relaying systems in terms of outage probability and error rates, there have been very limited contributions on ergodic capacity of fading relay channels [8, pp. 2286] (which may be attributed to the difficulty in evaluating the probability density function of the end-to-end SNR in closed-form) or explicitly highlighted the complexity of generalizing the MGF approach for channel capacity computation [9]. In [7], the authors circumvented this difficulty by evaluating upper and lower bounds on the capacity instead, while [8] resorted to the method of moments.

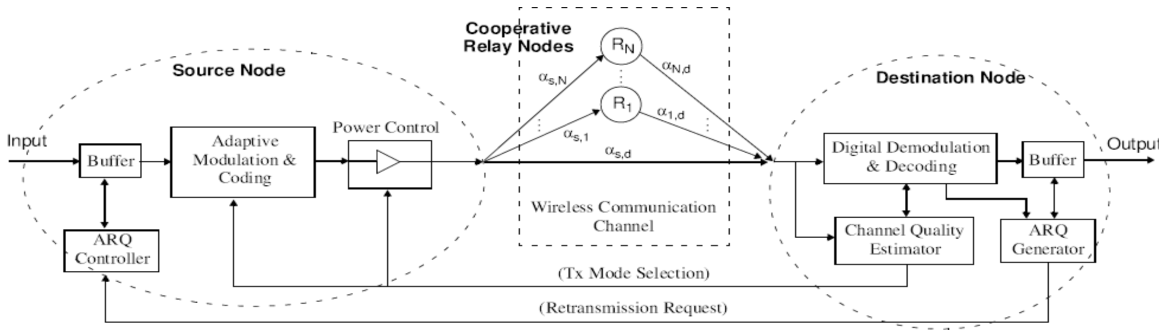

Fig. 1: Link-adaptive cooperative diversity system for ensuring the connectivity and network stability needed to support varying quality-of-service requirements in an airborne platform.

Nevertheless, both their analyses and results are limited to Rayleigh channels. In [10], the authors proposed a general method for channel capacity analysis using an integral relation known as E_i-transform, but their solution also requires the derivatives of the MGF and/or its auxiliary function, which can be cumbersome. Moreover, the capacity formula for the OPRA policy is an approximation while the capacity for the TCIFR policy was not considered. More recently, we have presented yet another unified approach for ergodic capacity computation based on a cumulative distribution function (CDF) method [11] (which utilizes the MGF of end-to-end SNR in conjunction with a fixed-Talbot method [12]).

II. System Model

In Fig. 1, the source node S communicates with destination node D via direct-link and through N amplify-and-forward relays, $R_i, i \in \{1, 2, ..., N\}$, in two transmission phases. During the initial Phase I, S broadcasts the signal x to D as well as to the relays R_i where the fading coefficients between S and D, S and the i-th relay node R_i, and R_i and D are denoted by $\alpha_{s,d}$, $\alpha_{s,i}$ and $\alpha_{i,d}$, respectively. The received signals at the destination and the relay is denoted respectively by

$$y_{s,d} = \sqrt{P_{s,d}}\,\alpha_{s,d} x + \eta_{s,d} \quad (1)$$

$$y_{s,i} = \sqrt{P_{s,i}}\,\alpha_{s,i} x + \eta_{s,i} \quad (2)$$

where $P_{s,d} = P_{s,i}$ is the transmitted power at source, while $\eta_{s,d}$, $\eta_{s,i}$ are the additive noises introduced between source to destination, and source to relay, respectively. During the second phase of cooperation, each of the N relays transmits the received signal after amplification via orthogonal transmissions (e.g., TDMA in a round-robin fashion and /or FDMA). Suppose $P_{i,d}$ is the transmit power of the i-th relay while transmitting its signal to the destination node. The received signal at the destination during the second phase of operation is given by [1]

$$y_{i,d} = G_i \alpha_{i,d} y_{s,i} + \eta_{i,d} \quad (3)$$

where G_i is the i-th relay amplifier gain, denoted as [1], $G_i = \sqrt{P_{i,d}} \Big/ \sqrt{P_{s,i}\left|\alpha_{s,i}\right|^2 + N_0}$, $\eta_{i,d}$ is the noise introduced between i-th relay and destination. We further assume that the total transmitted power $P_{s,d} + \sum_{i=1}^{N} P_{i,d} = P_T$ is fixed.

Suppose if a maximum-ratio combining (MRC) is employed at the destination node, the total received SNR can be shown as [1]

$$\gamma_T = \gamma_{s,d} + \sum_{i=1}^{N} \frac{\gamma_{s,i}\gamma_{i,d}}{1 + \gamma_{s,i} + \gamma_{i,d}} \cong \gamma_{s,d} + \sum_{i=1}^{N} \gamma_i = \gamma_{TB} \quad (4)$$

where $\gamma_{s,i} = \left|\alpha_{s,i}\right|^2 P_{s,i} T_s / N_0$ and $\gamma_{i,d} = \left|\alpha_{i,d}\right|^2 P_{i,d} T_s / N_0$ denote the instantaneous SNRs of the respective links, while the instantaneous SNR of a two-hops path can be accurately approximated as the harmonic mean of the link SNRs. The total SNR may be upper and lower bounded as

$$\gamma_{s,d} + \frac{1}{2}\sum_{i=1}^{N} \tilde{\gamma}_i = \gamma_{LB} \le \gamma_{TB} \le \gamma_{UB} = \gamma_{s,d} + \sum_{i=1}^{N} \tilde{\gamma}_i, \quad (5)$$

where $\tilde{\gamma} = \min(\gamma_{s,i}, \gamma_{i,d})$.

If $\gamma_{s,d}, \gamma_{s,i}, \gamma_{i,d}$ are independent random variates, then it is straight forward to show that the MGF of γ_{TB} is given by

$$\phi_{\gamma}(s) = \phi_{\gamma_{s,d}}(s) \prod_{i=1}^{N} \phi_{\gamma_i}(s) \quad (6)$$

For instance, the MGF of $\gamma_i = \gamma_{s,i}\gamma_{i,d} / (\gamma_{s,i} + \gamma_{i,d})$ for a Rayleigh channel with independent but non-identically distributed (i.n.d) fading statistics is well-known, and it is given by [14]

$$\begin{aligned}\phi_{\gamma_i}(s) &= \left[(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + (1/\Omega_{s,i} + 1/\Omega_{i,d})s\right]/\Delta^2 \\ &+ \frac{2s}{\Delta^3 \Omega_{s,i}\Omega_{i,d}} \ln\left(\left(s + \Delta + \frac{1}{\Omega_{s,i}} + \frac{1}{\Omega_{i,d}}\right)^2 \frac{\Omega_{s,i}\Omega_{i,d}}{4}\right)\end{aligned} \quad (7)$$

where $\Omega_{a,b} = E[\gamma_{a,b}]$ corresponds to the mean link SNR, and

$$\Delta = \sqrt{(1/\Omega_{s,i} - 1/\Omega_{i,d})^2 + 2s(1/\Omega_{s,i} + 1/\Omega_{i,d}) + s^2}$$

Moreover, the MGF of γ_i for a Nakagami channel with independent identically distributed (i.i.d) fading statistics is given by [15]

$$\phi_{\gamma_i}(s) = {}_2F_1\left(m, 2m; m + \frac{1}{2}; -\frac{\Omega s}{4m}\right) \quad (8)$$

If a closed-form expression for γ_i is not available (e.g., Nakagami-m channel with i.n.d fading statistics), but does exist for $\tilde{\gamma}_i = \min(\gamma_{s,i}, \gamma_{i,d})$, we may then develop capacity bounds using the inequality used in [7], (i.e., see (5)) viz.,

$$\phi_{\gamma_{s,d}}(s)\prod_{i=1}^{N}\phi_{\tilde{\gamma}_i}(s) \le \phi_{\gamma}(s) \le \phi_{\gamma_{s,d}}(s)\prod_{i=1}^{N}\phi_{\tilde{\gamma}_i}(s/2) \qquad (9)$$

In this case, the MGF of $\tilde{\gamma}_i$ for 2-hops path may be derived as

$$\phi_{\tilde{\gamma}_i}(s) = \sum_{\substack{k\in\{(s,i),(i,d)\}\\ j\neq k}} \int_0^\infty e^{-sx} f_{\gamma_k}(x)[1-F_{\gamma_j}(x)] \quad dx \qquad (10)$$

For instance, it is not difficult to show that the MGF of $\tilde{\gamma}_i$ for Nakagami-m fading with i.n.d fading statistics is given by

$$\phi_{\tilde{\gamma}_i}(s) = \sum_{\substack{k\in\{(s,i),(i,d)\}\\ j\neq k}} \frac{\Gamma(m_k+m_j)}{\Gamma(m_k)\Gamma(m_j)}\left(\frac{\Omega_j m_k}{s\Omega_j\Omega_k+\Omega_j m_k+\Omega_k m_j}\right)^{m_k} \qquad (11)$$

$$\times\ \frac{1}{m_k}\,{}_2F_1\left(1-m_j, m_k;\ 1+m_k; \frac{(s\Omega_k+m_k)\Omega_j}{s\Omega_j\Omega_k+\Omega_j m_k+\Omega_k m_j}\right)$$

Once the MGF of γ_i or $\tilde{\gamma}_i$ is found, we can compute the outage probability (i.e., it's CDF) efficiently using a fixed Talbot (i.e., multi-precision Laplace transform inversion) method [12], viz.,

$$F_x(x) \cong \frac{r}{M}\sum_{k=1}^{M-1}\mathrm{Re}\{e^{xs(\theta_k)}\phi(s(\theta_k))\frac{1+j\sigma(\theta_k)}{s(\theta_k)}\} + \frac{1}{2M}\phi_x(r)e^{rx}, \qquad (12)$$

where $r = 2M/(5x)$, $\sigma(\theta_k) = \theta_k + (\theta_k\cot(\theta_k)-1)\cot(\theta_k)$, $\theta_k = k\pi/M$ and $s(\theta_k) = r\theta_k(j+\cot(\theta_k))$.

In Section III, we will show that the ergodic capacity of ORA, TCIFR, and OPRA source adaptive transmission policies can be expressed in terms of the MGF alone. Hence the ergodic capacity may be evaluated readily for all of the cases discussed above (since the MGF of γ_i or $\tilde{\gamma}_i$ is available in closed-form). For instance, (15), (17) and (22) in conjunction with (6), (9), (11) and (12) generalize the results in [7] to i.n.d Nakagami-m channels. Similarly, precise estimates of the ergodic capacities with different adaptive source transmission techniques in i.n.d Rayleigh fading [or i.i.d Nakagami-m fading] can be obtained by substituting (6) and (7) [or (8)] into (15), (17) and (22).

III. Ergodic Capacity Computation in Fading Channels

The well-known Shannon-Hartley law tells us that there is an absolute limit on the error-free bit rate R that can be transmitted within a certain channel bandwidth B at a specified SNR. This theoretical limit denotes the channel capacity C. Shannon's noisy channel coding theorem also states that, it is not possible to make the probability of error tend to zero if $R > C$ with any code design. Thus, it is clear that the metric C plays an important role in the design or appraisal of any communications system (since it serves as an upper limit on the transmission rate for reliable communications over a noisy communication channel). In this section, we will derive generic expressions for computing the ergodic capacity with different adaptive source transmission techniques.

A. ORA Policy

When only the rate is adapted by changing channel conditions, the channel capacity is given by [6]

$$\frac{\bar{C}_{ORA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \ln(1+\gamma) f_\gamma(\gamma)d\gamma \qquad (13)$$

Utilizing the "exponential type" integral representation of $\ln(\gamma+1)$ (see Appendix A), we can facilitate the averaging problem in (13) given that the MGF $\phi_\gamma(\cdot)$ is available in closed form. Substituting (A.5) into (13), we obtain

$$\frac{\bar{C}_{ORA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \frac{e^{-2x}}{x}\left[\int_0^\infty (1-e^{-2x\gamma}) f_\gamma(\gamma)d\gamma\right]dx = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \frac{e^{-2x}}{x}\left[1-\phi_\gamma(2x)\right]dx \qquad (14)$$

Substituting $y = 2x, dy = 2dx$, (14) can be re-stated as

$$\frac{\bar{C}_{ORA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^\infty \frac{e^{-y}}{y}\left[1-\phi_\gamma(y)\right]dy \qquad (15)$$

Note that $\phi_\gamma(\cdot)$ is a monotonically decreasing function with respect to its argument and $0 \le \phi_\gamma(s) \le 1$.

B. TCIFR Policy

In CIFR policy, the transmitter adapts its power to maintain a constant SNR at the receiver and uses fixed-rate modulation and fixed-code designs. This technique is the least complex to implement given that reliable channel estimates are available at the transmitter. However, when the channel experiences deep fades, the penalty in transmit power requirement with the CIFR policy will be enormous because channel inversion needs to compensate for the deep fades. To overcome this, a truncated channel inversion and fixed rate policy (TCIFR) was considered in [6] where the channel fading is only inverted above a fixed cutoff fade depth γ_0. The data transmission is ceased, if γ falls below γ_0. In this case, it is easy to show that the outage probability is $P_{out} = F_\gamma(\gamma_0)$ and the channel capacity is given by [6]

$$\frac{\bar{C}_{TCIFR}}{B} = \frac{1}{N+1}\log_2\left(1+\frac{1}{\int_{\gamma_0}^\infty \gamma^{-1} f_\gamma(\gamma)d\gamma}\right)F_\gamma^c(\gamma_0) \qquad (16)$$

Substituting $f_\gamma(\gamma) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\phi_\gamma(-j\omega)e^{-j\omega\gamma}\right]d\omega$ (i.e., inverse Fourier Transform of the characteristic function) in (16), we obtain

$$\frac{\bar{C}_{TCIFR}}{B} = \frac{1}{N+1}\log_2\left(1-\frac{1}{\nabla}\right)F_\gamma^c(\gamma_0) \qquad (17)$$

where $\nabla = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\{\phi_\gamma(-j\omega)Ei(-j\omega\gamma_0)\}d\omega$ while exponential integral $Ei(-jc) = \int_{-1}^\infty \frac{e^{-jct}}{t}dt$ can be evaluated using MATLAB as $\cos\mathrm{int}(c) - j(\pi/2+\sin\mathrm{int}(c))$. Observe that, if the MGF of γ is known in closed-form, then the above integral can be

evaluated efficiently via Gauss-Chebyshev quadrature method over a wide-range of fading channel models and diversity combining techniques employed.

C. OPRA Policy

In OPRA scheme, the transmission power and rate is matched to the varying channel condition through use of a multiplexed multiple codebook design. This leads to the highest achievable capacity with CSI. From [6], we have

$$\frac{\bar{C}_{OPRA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_{\gamma_0}^{\infty} \ln\left(\frac{\gamma}{\gamma_0}\right) f_\gamma(\gamma)d\gamma \qquad (18)$$

Substituting (A.4) into (18), and by rearranging the integral terms we obtain,

$$\frac{\bar{C}_{OPRA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty} \frac{1}{x}\left[e^{-2x}\int_{\gamma_0}^{\infty} f_\gamma(\gamma)d\gamma - \int_{\gamma_0}^{\infty} e^{\frac{-2x\gamma_0}{\gamma_0}} f_\gamma(\gamma)d\gamma\right]dx \qquad (19)$$

which can be re-stated as

$$\frac{\bar{C}_{OPRA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty} \frac{1}{x}\left[e^{-2x}[1-F_\gamma(\gamma_0)] - \phi_\gamma(2x/\gamma_0, \gamma_0)\right]dx \qquad (20)$$

since $\int_{\gamma_0}^{\infty} f_\gamma(\gamma)d\gamma = 1 - F_\gamma(\gamma_0) = F_\gamma^c(\gamma_0)$ is the complementary CDF of γ and $\int_{\gamma_0}^{\infty} e^{\frac{-2x\gamma}{\gamma_0}} f_\gamma(\gamma)d\gamma = \phi_\gamma(2x/\gamma_0, \gamma_0)$ denotes the marginal MGF. Since the desired marginal MGF is typical not available in closed-form, we circumvent this difficulty by computing this term via the CDF of an auxiliary function (details shown in Appendix B), viz.,

$$\frac{\bar{C}_{OPRA}}{B} = \frac{1}{N+1}\frac{1}{\ln 2}\int_0^{\infty} \frac{1}{x}\left[e^{-2x}[1-F_\gamma(\gamma_0)] - \phi_\gamma\left(\frac{2x}{\gamma_0}\right) + F_{\hat{\gamma}}(\gamma_0)\right]dx \qquad (21)$$

In (21), $\phi_\gamma(2x/\gamma_0)$ is available in the closed form while the CDF terms $F_\gamma(\gamma_0)$ and $F_{\hat{\gamma}}(\gamma_0)$ are compute efficiently using the fixed-Talbot method (see (12)).

To achieve the capacity shown in (21), the channel fade level (i.e., CSI) tracked at the receiver must be conveyed to the transmitter on the feedback path for power and rate adaptation in accordance with the prevailing channel condition. When $\gamma < \gamma_0$, no data is transmitted, and thus the optimal policy suffers an outage probability given by $P_{out} = F_\gamma(\gamma_0)$, which equals to the probability of no transmission. The optimal cutoff SNR must satisfy

$$F_\gamma^c(\gamma_0) - \gamma_0[1 + \int_{\gamma_0}^{\infty} \gamma^{-1} f_\gamma(\gamma)d\gamma] = 0 \qquad (22)$$

The integral term in (22) can be evaluated efficiently similar to the development of (17) when the MGF is available. Furthermore, asymptotic analysis of (22) shows that $\gamma_0 \to 0$ when the mean SNR $\Omega \to 0$ because $F_\gamma(x) \to 1$ and $f_\gamma(x) \to 0$ (the effect of $\Omega \to 0$ can be predicted from the normalized PDF or the normalized CDF curve when its argument $x \to \infty$). When $\Omega \to \infty$, $F_\gamma(x) \to 0$ because this is equivalent to computing the CDF when its argument $x \to 0$. It is also well-known that $\phi_\gamma(\cdot) \to 0$ as $\Omega \to \infty$. Hence, $\gamma_0 \to 1$ as $\Omega \to \infty$. Thus γ_0 (can be determined by solving (22) numerically) always lies in the interval [0, 1] regardless of the fading channel model and number of relay nodes employed.

IV. Computational Results

In this section, selected numerical results are presented for the Shannon capacities of cooperative analog relaying systems under different source adaptive transmission policies. The following parameters (arbitrarily chosen) will be used to generate the plots, unless stated otherwise: $\Omega_{s,1} = E_s/N_0$, $\Omega_{s,2} = 0.5E_s/N_0$, $\Omega_{1,d} = 0.5E_s/N_0$, $\Omega_{2,d} = E_s/N_0$, and $\Omega_{s,d} = 0.2E_s/N_0$.

Fig. 2 depicts the comparison channel capacities of three distinct source adaptive transmission policies in an i.n.d Rayleigh fading channels with two cooperative relays. As anticipated, there is no significant difference observed in the capacities of OPRA and ORA at high SNR. The capacity of TCIFR is plotted for the cut-off SNR γ_0 = 6 dB. Although not shown in this figure, we also noticed that the curves corresponding to the tight-bound case are in good agreement with the Monte Carlo simulation results. Moreover, the actual capacity is slightly closer to the lower bound (rather than the upper bound) at low SNRs. Although the authors in [7] have studied the channel capacities of cooperative relaying system in i.n.d Rayleigh fading channel, but their framework does lend itself to the analysis of the "tight-bound" case or generalize to other fading channels, whereas our framework encapsulates all these cases in an unified way.

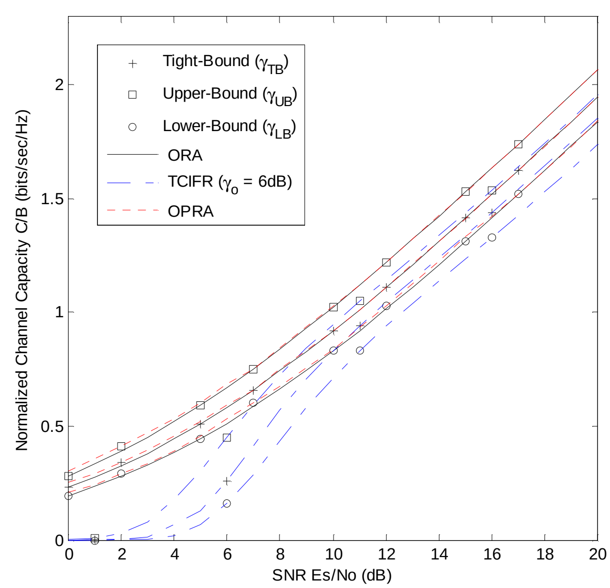

Fig. 2 Channel capacities of ORA, OPRA and TCIFR in an i.n.d Rayleigh fading channel with two cooperative relays.

Fig 3 illustrates the TCIFR channel capacities for E_s/N_0 = 6dB and E_s/N_0 = 15dB as a function of cut-off SNR γ_0 in an i.n.d Rayleigh fading channel with two relays. It is evident that there exists an optimal choice of the cut-off SNR which will maximize the channel capacity while E_s/N_0 is fixed.

Fig.4 shows the channel capacities (i.e., the tight-bound case) for different source adaptive transmission schemes over i.i.d Nakagami-m channels (fading severity index m = 0.5, 1, 1.5 and 2). It is apparent that the channel capacity increases with the increasing value of m (i.e., as channel experiences

less severe fading). Moreover, the gap between the ORA and OPRA capacities widens as m decreases.

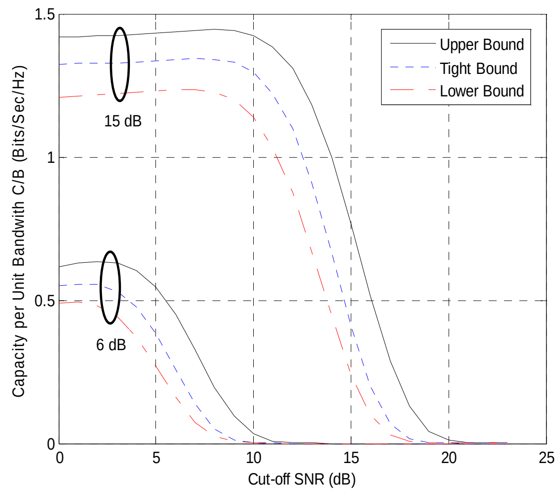

Fig. 3 TCIFR channel capacity versus cut-off SNR in an i.n.d Rayleigh fading channel with two relays.

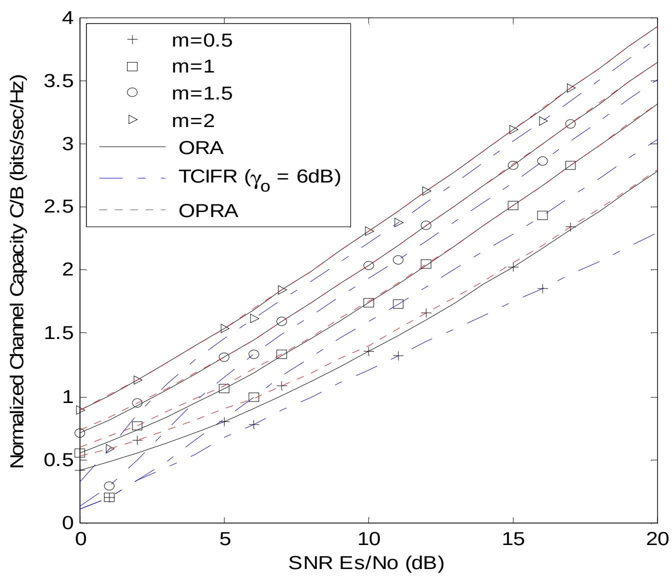

Fig. 4 Channel capacities of ORA, OPRA and TCIFR policies in an i.i.d Nakagami-m channel with a single cooperative relay.

Fig. 5 depicts the ORA channel capacity for i.n.d Nakagami-m channels with varying number of relays (N = 0, 1, 2) and transmit power allocation. Without any loss of generality, we can show that the average SNRs on different links are given as follows:

$$\Omega_{s,1} = \left(\frac{d_{s,1}}{d_{s,d}}\right)^{-n} c\,\delta_{s,1}\, E_T/N_o\,,\ \Omega_{s,2} = \left(\frac{d_{s,2}}{d_{s,d}}\right)^{-n} c\,\delta_{s,2}\, E_T/N_o\,,$$

$$\Omega_{1,d} = \left(\frac{d_{1,d}}{d_{s,d}}\right)^{-n} c\,\delta_{1,d}\, E_T/N_o\,,\ \Omega_{2,d} = \left(\frac{d_{2,d}}{d_{s,d}}\right)^{-n} c\,\delta_{2,d}\, E_T/N_o$$

and $\Omega_{s,d} = \left(\frac{d_{s,d}}{d_{s,d}}\right)^{-n} c\,\delta_{s,d}\, E_T/N_o$, where c is constant that is related to the carrier wavelength (i.e., $c = \left(\frac{\lambda}{4\pi}\right)^2$ for free space path loss), n is the path loss exponent, $\delta_{s,d} = \delta_{s,i} = \frac{P_{s,i}}{P_T}$ while $\delta_{i,d} = (1-\delta_{s,i}) = \frac{P_{i,d}}{P_T}$ since $\sum \delta_{a,b} = 1$, and $d_{a,b}$ denotes the distance between link a-b. To generate Fig. 5, we have arbitrarily chosen $d_{s,1} = d_{s,2} = d_{1,d} = d_{2,d} = 500\,m$, $c = 10^{-2}$, $d_{s,d} = 1000\,m$, and $n = 4$.

It is evident that equal power allocation strategy may be reasonable but not optimal in all cases (i.e., optimum power allocation strategy is more beneficial when the channel experience more severe fading). It is also observed that for the chosen set of parameters, increasing the number of cooperating relays beyond a certain limit (in this case, N = 1) does not necessarily translates into higher spectral efficiency (although diversity gain may be harnessed) – there exists an optimum number of cooperating nodes. For instance, the single relay case yields the highest capacity, followed by the two relays case, and finally the no relay case for values of $\delta_{s,i}$ below 0.9 when m = 3 and E_T/N_0 = 20dB. We also observe that the tightness of upper and lower bounds on the Shannon capacity becomes looser as m increases. By comparing the curves corresponding to the single relay and no relay cases, we can deduce that the optimum power allocation is not strongly influenced by the source-destination link but rather on the link qualities between source-relay and relay-destination.

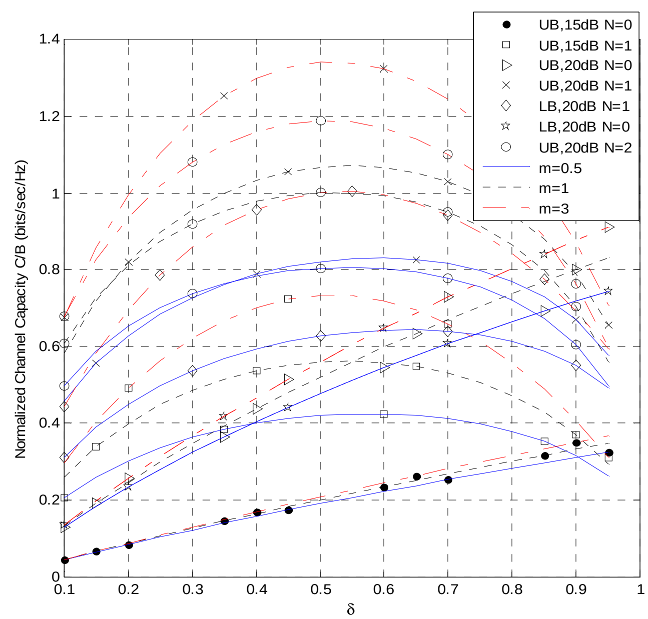

Fig. 5 Shannon capacity of a cooperative relaying system consisting of N relays with ORA policy but with different fixed transmit power allocations in a i.n.d Nakagami-m channels.

APPENDIX A

In this appendix, we provide a sketch of our derivation for an "exponential-type" integral representation for $\ln\gamma$. Such a representation will facilitate the averaging problem that arises in the channel capacity analysis, and therefore leads to a unified approach for evaluating the ergodic Shannon capacity in a myriad of fading environments and relaying strategies.

Utilizing [13, eq. (1.512.2)], we have

$$\ln\gamma = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}\left(\frac{\gamma-1}{\gamma+1}\right)^{k} = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}(y)^{k},\quad \gamma>0 \tag{A.1}$$

where $y=\dfrac{\gamma-1}{\gamma+1}$. Substituting $y^{k}=\dfrac{1}{\Gamma(k)}\int_0^\infty x^{k-1}e^{-x/y}\,dx$ [13, eq. (3.381.4)] into (A.1), we obtain

$$\ln\gamma = 2\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k}\left(\frac{1}{\Gamma(k)}\int_0^\infty x^{k-1}e^{-x/y}\right)dx = 2\int_0^\infty e^{-x/y}\left(\sum_{\substack{k=1\\k\ odd}}^{\infty}\frac{1}{k!}x^{k-1}\right)dx \tag{A.2}$$

Recognizing that $\dfrac{1}{x}\ sh\ x\ =\dfrac{e^{x}-e^{-x}}{x}=\sum_{\substack{k=1\\k\ odd}}^{\infty}\dfrac{1}{k!}x^{k-1}$ [13, Eq. (1.411.2)], (A.2) can be re-stated as

$$\ln\gamma\ = 2\int_0^\infty e^{-x/y}\frac{1}{x}(e^{x}-e^{-x})e^{-x\left(\frac{\gamma+1}{\gamma-1}\right)}dx \tag{A.3}$$

Finally using variable substitution $x=z(\gamma-1),\ dz=\dfrac{dx}{\gamma-1}$, we arrive at (A.4) after some routine algebraic manipulations:

$$\ln\gamma = \int_0^\infty \frac{1}{z}\left[\ e^{-2z}-e^{-2z\gamma}\right]dz,\quad \gamma>0 \tag{A.4}$$

It is also obvious that

$$\ln(\gamma+1) = \int_0^\infty \frac{e^{-2z}}{z}\left[\ 1-e^{-2z\gamma}\right]dz,\quad \gamma>\text{-}1 \tag{A.5}$$

APPENDIX B

Let $\phi_X(s)=\int_0^\infty e^{-sx}f_X(x)dx$ and $\Phi_X(j\omega)=\int_0^\infty e^{j\omega x}f_X(x)dx$ denote the MGF and the characteristic function (CHF) of random variable $X\geq 0$ respectively. Thus the CHF is related to the MGF as $\Phi_X(j\omega)=\phi_X(-j\omega)$.

Also while evaluating the ergodic capacity for OPRA policy, we must compute the marginal MGF of SNR. Suppose the MGF $\phi_\gamma(\cdot)$ is known in closed, we may then use Abate's fixed Talbot method (multi-precision Laplace inversion) for computing the marginal MGF efficiently. A sketch of this derivation is highlighted next:

Let us define an auxiliary function $f_{\hat\gamma}(x)=e^{-\beta x}f_\gamma(x)$. Hence,

$$\phi_\gamma(\beta,\alpha)=\int_\alpha^\infty e^{-\beta\gamma}f_\gamma(\gamma)d\gamma = F_{\hat\gamma}(\infty)-F_{\hat\gamma}(\alpha) \tag{B.1}$$

where $F_{\hat\gamma}(y)=\int_0^y f_{\hat\gamma}(x)dx=\int_0^y e^{-\beta x}f_\gamma(x)dx$. It is obvious that

$$F_{\hat\gamma}(\infty)=\int_0^\infty e^{-\beta x}f_\gamma(x)dx=\phi_{\hat\gamma}(0)=\phi_\gamma(\beta) \tag{B.2}$$

Furthermore, we have

$$\begin{aligned}\phi_{\hat\gamma}(s) &= \int_0^\infty e^{-sx}f_{\hat\gamma}(x)dx,\\ &= \int_0^\infty e^{-(s+\beta)x}f_{\hat\gamma}(x)dx,\\ &= \phi_\gamma(s+\beta)\end{aligned} \tag{B.3}$$

Substituting (B.2) and (B.3) (in conjunction with (12)) into (B.1), we arrive at a numerically efficient method for evaluating the required marginal MGF.

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